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A Modal Logic of Metaphor

Dedicated to Professor Ryszard Wójcicki
on the occasion of his 80th birthday

Abstract. The purpose of this paper is to suggest a formal modelling of metaphors as a lingustic tool capable of conveying meanings from one conceptual space to another. This modelling is done within DDL (dynamic doxastic logic).

Keywords: Dynamic doxastic logic, modal logic, pragmatic theory of belief.

If the only function of metaphor had been to convey information it is unlikely that it would have been used so often by poets, dramatists, novelists, orators, politicians and many others, not least ordinary people. Metaphor has many functions, and there are many reasons why scholars should be interested in metaphor. Having said this, let us now immediately forget about all those other aspects of metaphor and concentrate on the purely descriptively informative one.

The question that interests us here is this: How come we are interested in statements that, if treated on the level of ordinary statements, are just nonsense? The best answer may be another question: What do you do if you want to communicate something by linguistic means only—an experience, a feeling, a sensation, an idea—and you find that your language is not up to it? Perhaps the task is impossible: *wovon man nicht sprechen kann, darüber muss man schweigen.*[1]

Or perhaps not. Wittgenstein's famous dictum concerns human language pushed to the limit. At more modest, everyday levels the situation is different: there language may still be capable of meaningful growth. After all, if human experience is increasing all the time, individually as well as collectively, then it is not so surprising that the linguistic means to express this experience should sometimes be lagging. And it is also not so surprising that one should try to express new experience as best one can by resorting

[1] "Whereof one cannot speak, thereof one must be silent."

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to whatever language one already possesses.[2] In particular, one might try to use concepts belonging to what may loosely be called one conceptual space in order to describe what is going on in another conceptual space.[3] This is where metaphor comes in: *wovon man nicht sprechen kann, dafür kann man vielleicht immerhin eine Metapher finden.*[4]

The purpose of this paper is to suggest a formal modelling explaining how such growth may be possible. We do this within the frame-work of DDL (dynamic doxastic logic), a modal logic of belief change that extends AGM, the theory of belief change developed in the 1980s by Carlos Alchourrón, Peter Gärdenfors and David Makinson (for AGM see [1]; for DDL see [4, 6, 7, 8, 9]).

The idea that metaphors can be used to transmit information and to influence the factual beliefs of an audience is surely not unnatural. The starting point is the assumption that what a rational agent believes at a certain time can be represented by a certain "belief set"; that is, the set of beliefs actually held by the agent at that time. Even more: there is in fact a "belief system", an entire family of similar sets, each set representing a set of possible beliefs—sets which become important if the agent accepts new information requiring that currently existing beliefs be modified.

As explained in section 1 we will use formulæ of type $\mathbf{B}\phi$ and $\mathbf{K}\phi$ with informal readings "the agent believes that ϕ" (that is, he currently believes that) and "the agent is committed to the belief that ϕ" (that is, whatever happens he will never give up his belief that ϕ). Furthermore, there is a dynamic operator $*$ representing revision of belief: $[*\phi]\theta$ stands for "after the agent has revised his beliefs by accepting that ϕ, it is the case that θ".

In section 2 we will then go on to present a rudimentary metaphoric logic. Here we will use a letter, M say, to name a certain metaphor, while an expression of type $\binom{\mathsf{M}}{\phi}$, where ϕ is an ordinary ("literal") formula, may be read "ϕ under M".

No formal theorem is proved. The main interest is focussed on how a metaphoric expression may generate a new model from the old model, the new model being in a certain sense more complete than the old one. It is also suggested that this is a way of "killing" a metaphor: a metaphor that is "alive" in the old model is "dead" in the new one.

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[2] Think of how one behaves when visiting another country with a language of which one's command is painfully inadequate but not totally nonexistent. With enough creativity from both visitor and indigenous a degree of communication is usually possible.

[3] For one conception of conceptual space, see [2].

[4] "For that of which one cannot speak, perhaps one can yet find a metaphor."

We give no examples of metaphors in this paper, instead referring readers to the standard work by George Lakoff and Mark Johnson [3]. For a delightful survey of figures of speech, see Quinn [5].

1. Dynamic doxastic logic

We begin by reproducing a version of AGM as translated into the language of DDL.

1.1. Formal language

We assume a propositional language with denumerably many propositional letters, enough Boolean operators, two doxastic operators $\mathbf{B}$ and $\mathbf{K}$, and a special operator $*$. A *pure Boolean* formula is one formed from propositional letters with the help of Boolean operators only. A *static doxastic* formula is one formed with the help of Boolean operators from pure Boolean formulæ and formulæ of the form $\mathbf{B}\phi$ or $\mathbf{K}\phi$, where ϕ is pure Boolean. A *dynamic doxastic formula* is one formed with the help of Boolean operators from static formulæ and formulæ of the form $[*\phi]\theta$, where ϕ is pure Boolean and θ is static doxastic. (Since the dynamic doxastic formulæ constitute the most general category of formulæ in this paper, we normally refer to them just as *formulæ*. This is a fairly restrictive object language, but for the purposes of this article the restriction seems reasonable.)

Formal syntax

We build an axiom system in three steps, dividing the postulates into classical, general, and special.

Postulates for classical logic: all tautologies plus modus ponens.

General postulates: all instances of the following schemata (where ϕ and ψ must be pure Boolean)

(K**B**) $\mathbf{B}(\phi \to \psi) \to (\mathbf{B}\phi \to \mathbf{B}\psi)$

(K**K**) $\mathbf{K}(\phi \to \psi) \to (\mathbf{K}\phi \to \mathbf{K}\psi)$

(K$*$) $[*\phi](\theta \to \varpi) \to ([*\phi]\theta \to [*\phi]\varpi)$

as well as the following rules (where ϕ and ψ must be pure Boolean)

(N**B**) if θ is a thesis, then so is $\mathbf{B}\theta$, (N**K**) if θ is a thesis, then so is $\mathbf{K}\theta$,

(N$*$) if θ is a thesis, then so is $[*\phi]\theta$,

(E$*$) if $\phi \leftrightarrow \psi$ is a thesis, then so is $[*\phi]\theta \leftrightarrow [*\psi]\theta$.

Special postulates: All well-formed instances of the following axiom schemata:

($*$2) $[*\phi]\mathbf{B}\phi$

($*$3) $[*\top]\mathbf{B}\theta \rightarrow \mathbf{B}\theta$

($*$4) $\mathbf{b}\top \rightarrow (\mathbf{B}\theta \rightarrow [*\top]\mathbf{B}\theta)$

($*$5) $\mathbf{b}\top \rightarrow (\mathbf{k}\phi \rightarrow [*\phi]\mathbf{b}\top)$

($*$6) $\mathbf{K}(\phi \leftrightarrow \psi) \rightarrow ([*\phi]\theta \leftrightarrow [*\psi]\theta)$

($*$7) $[*(\phi \wedge \psi)]\mathbf{B}\theta \rightarrow [*\phi]\mathbf{B}(\psi \rightarrow \theta)$

($*$8) $\langle*\phi\rangle\mathbf{b}\psi \rightarrow ([*\phi]\mathbf{B}(\psi \rightarrow \theta) \rightarrow [*(\phi \wedge \psi)]\mathbf{B}\theta)$

($*$D) $[*\phi]\theta \rightarrow \langle*\phi\rangle\theta$

($*$FB) $\langle*\phi\rangle\mathbf{B}\theta \rightarrow [*\phi]\mathbf{B}\theta$

($*$X) $\theta \leftrightarrow [*\phi]\theta$ (if θ is a pure Boolean formula)

($*$KX) $\mathbf{K}\theta \leftrightarrow [*\phi]\mathbf{K}\theta$

Here we are assuming the following definitions:

(df**b**) $\mathbf{b}\phi =_{\mathrm{df}} \neg\mathbf{B}\neg\phi$

(df**k**) $\mathbf{k}\phi =_{\mathrm{df}} \neg\mathbf{K}\neg\phi$

(df$\langle\cdot\rangle$) $\langle\phi\rangle\theta =_{\mathrm{df}} \neg[\phi]\neg\theta$

As usual, a *derivation* or *formal proof* is a sequence of formulæ in which every member is an axiom or follows from previous formulæ by one of the inference rules. A *thesis* or *formal theorem* is a formula which can appear as the last formula in a formal proof.

It is worth adding a brief comment on the axiom system just defined. ($*$2)–($*$8) are—or, in the case of ($*$3) and ($*$4), are equivalent to—DDL versions of the original AGM-postulates, while ($*$D), ($*$FB), ($*$X) and ($*$KX) correspond to conditions that are, or seem to be, implicit in the original formulation of AGM. (The original numbering in [1] has been kept. There is no need for a schema ($*$1) here since the corresponding condition is automatically satisfied.)

1.2. Formal semantics

A *frame* is a structure (U, T, P, Q, R) such that

(i) (U, T) is a compact, totally separable topological space,[5]

(ii) P is the set of clopen subsets of U,

(iii) Q is a set of sphere systems. (A sphere system in (U, T)—or, more colloquially, an "onion"—is a set of closed subsets of U which is (i) linearly ordered by set inclusion, (ii) closed under nonempty intersection.)

(iv) R is a function which assigns, for each clopen subset X or U, a binary relation on R_X on Q subject to the following conditions:

  (a) R_X is serial; that is, for each u there is some v such that $(u, v) \in R_X$;

  (b) R_X is functional; that is, if $(u, v) \in R_X$ and $(u, w) \in R_X$, then $v = w$;

  (c) R_X is appropriate; that is, if $\bigcup O \cap X \neq \emptyset$, then $(O, O') \in R_X$ only if $\bigcap O' = Z \cap X$, where Z is the smallest element of O to intersect X.

The last definition does not cover all cases: what if $\bigcup O \cap X = \emptyset$? Three possibilities come to mind, corresponding to three different kinds of agent:

$O' = O$. In this case there is no reaction on the part of the agent: the new information is rejected. Call this the case of the *staunch conservative.*

$O' = \{\bigcup O\}$. An agent reacting in this way accepts the new information and bites the bullet: all beliefs are given up that are not logically implied by his doxastic commitments. This agent may be called a *person of principle.*

$O' = \emptyset$. This is the case of an agent who gives up and who from now on accepts everything, including logical contradiction. Having given up, this agent is now *totally gullible.*

A *valuation* in a frame (U, T, P, Q, R) is a function from the set of propositional letters to the set P of clopen subsets of U. A *model* is a frame with a valuation in that frame.

Before going on to a definition of truth let us consider the following informal remark. The total situation an agent finds himself in consists of two components: beliefs and facts—on the one hand what he thinks is the case, and on the other what really is the case. Thus if O is an onion, representing the agent's beliefs, and u is an element of U, representing the real state-of-affairs, then the ordered pair (O, u) represents a total situation.

[5] For technical concepts not explained here, see [4].

We will now define the notion of a well-formed formula ϕ being *true* in a total situation *(O, u) in* a model $\mathfrak{M}$ with universe U, in symbols $(O, u) \vDash^{\mathfrak{M}} \phi$. The definition proceeds in two steps: first we define the meaning of pure Boolean formulæ, and then we extend that definition to the set of all formulæ.

First step: Let ϕ be a pure Boolean formula. If ϕ is a propositional letter, then the value $[\![\phi]\!]$ is defined by the nameless valuation of $\mathfrak{M}$. If ϕ is a complex formula, it is evaluation as follows:

$$[\![\neg\theta]\!] = U - [\![\theta]\!],$$
$$[\![\theta \wedge \varpi]\!] = [\![\theta]\!] \cap [\![\varpi]\!],$$

and similar conditions for other Boolean connectives.

Second step:

$(O, u) \vDash^{\mathfrak{M}} \phi$ iff $u \in [\![\phi]\!]$, if ϕ is a pure Boolean formula,

$(O, u) \vDash^{\mathfrak{M}} \neg\phi$ iff not $(O, u) \vDash^{\mathfrak{M}} \phi$,

$(O, u) \vDash^{\mathfrak{M}} \phi \wedge \psi$ iff $(O, u) \vDash^{\mathfrak{M}} \phi$ and $(O, u) \vDash^{\mathfrak{M}} \psi$,

and similar conditions for other Boolean operators;

$(O, u) \vDash^{\mathfrak{M}} \mathbf{B}\phi$ iff $\bigcap O \subseteq [\![\phi]\!]$,

$(O, u) \vDash^{\mathfrak{M}} \mathbf{K}\phi$ iff $\bigcup O \subseteq [\![\phi]\!]$,

$(O, u) \vDash^{\mathfrak{M}} [*\phi]\theta$ iff, for all onions O' such that $(O, O') \in R_{[\![\phi]\!]}$, it is the case that $(O', u) \vDash \phi$.

This ends the second step.

If it is not the case that $(O, u) \vDash^{\mathfrak{M}} \phi$ then we say that ϕ is *false* in (O, u) in $\mathfrak{M}$. We say that a formula ϕ is *true in* a model $\mathfrak{M}$, in symbols $\mathfrak{M} \vDash \phi$, if ϕ is true in every total situation in $\mathfrak{M}$; and we say that ϕ is *not true in* $\mathfrak{M}$ if ϕ is false in $\mathfrak{M}$ in at least one total situation.

Furthermore, we say that ϕ is *valid in* $\mathfrak{M}$ if ϕ is true in all total situations and under all valuations in $\mathfrak{M}$; that ϕ is *valid in* $\mathfrak{C}$ if $\mathfrak{C}$ is a class of frames and ϕ is true in all the frames belonging to $\mathfrak{C}$; and that ϕ is *generally valid* if valid in all frames.

There is the following well-known completeness result:

THEOREM. *A formula is provable in the given axiom system if and only if it is generally valid.*

2. Metaphoric logic

For obvious reasons, the material in this section is of a much more tentative nature. Nevertheless, we keep the format of the preceding section, collecting our remarks under the headings of (1) language, (2) syntax and (3) semantics.

2.1. Formal language

The formal language defined in the previous section is augmented with a number of *metaphoric letters* as well as a new two-place operator $\binom{\cdot}{\cdot}$. We sketch a definition of a new extended formal language. Formulæ made up of propositional letters and Boolean connectives are called *literal*. In addition we assume that there is a set of *metaphoric formulæ*; that is, Boolean combinations of certain formulæ of the form $\binom{\mathsf{M}}{\phi}$, where M is a metaphoric letter and ϕ is a pure Boolean formula. To make this precise: we assume that for each metaphoric letter M there is a set Δ_{M} of literal formulæ (the "domain" of the metaphor) such that the expression $\binom{\mathsf{M}}{\phi}$ is well-formed if and only if $\phi \in \Delta_{\mathsf{M}}$. However, we impose no closure conditions on Δ_{M}.[6]

A *basic* formula is one that is a Boolean combination of literal and metaphoric formulæ. A *general* formula is one that is a Boolean combination of formulæ, each of which is of the form $\mathbf{B}\phi$ or $\mathbf{K}\phi$ or $[*\phi]\theta$, where ϕ is basic (while θ may be any formula).

The intuitive idea is that every metaphor has a certain "halo effect". It is this feature that explains why an aptly chosen metaphor can be understood also by those who have never heard it before and who might not be able to articulate its meaning in their own language. That is to say, for every metaphor there is a certain set of propositions which it makes sense to view in a light different from that it which it would normally be viewed; the meaning of those propositions under that metaphor (their metaphorical meaning) is different from the meaning they normally have (their literal meaning).

In somewhat more technical language (but still vague), for every successful metaphor M and appropriate literal formula ϕ there is a non-empty set $\nabla(\mathsf{M}, \phi)$ of ordinary (literal) formulæ which is in some sense equivalent to ϕ under M; this set we shall call the *halo of ϕ under M*. The idea is that interpreting a proposition ϕ under a metaphor M amounts to treating the

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[6]The domains of a metaphors vary considerably. See [3] for a number of detailed examples.

information contained in the single expression $\binom{\mathsf{M}}{\phi}$ as logically equivalent to the information conveyed by the entire set $\nabla(\mathsf{M}, \phi)$.

2.2. Formal syntax

The axiom system in the preceding section is generalized by expanding the range of the axiom schemata to the present extended notion of formal language. Thus, for example, (assuming well-formed expressions)

$$[*\binom{\mathsf{M}}{\phi}](\theta \to \varpi) \to ([*\binom{\mathsf{M}}{\phi}]\theta \to [*\binom{\mathsf{M}}{\phi}]\varpi)$$
$$[*\binom{\mathsf{M}}{\phi}]\mathbf{B}\binom{\mathsf{M}}{\phi}$$

are instances of the extended schemata (K$*$) and ($*$2), respectively. To take another example (again assuming well-formed expressions)

$$[*(\binom{\mathsf{M}}{\phi} \wedge \psi)]\mathbf{B}\theta \to [*\binom{\mathsf{M}}{\phi}]\mathbf{B}(\psi \to \theta)$$
$$[*(\phi \wedge \binom{\mathsf{M}}{\psi})]\mathbf{B}\theta \to [*\phi]\mathbf{B}(\binom{\mathsf{M}}{\psi} \to \theta)$$

are instances of the extended schema ($*$7). There are also the following two new rules (assuming, in the first case that $\binom{\mathsf{M}}{\phi}$ is well-formed, and in the second case that $\binom{\mathsf{M}}{\phi}$ and $\binom{\mathsf{M}}{\psi}$ are well-formed):

(NM) If θ is a thesis, then so is $[*\binom{\mathsf{M}}{\phi}]\theta$.

(EM) If $\phi \leftrightarrow \psi$ is a thesis, then so is $\binom{\mathsf{M}}{\phi} \leftrightarrow \binom{\mathsf{M}}{\psi}$.

Furthermore—and this is the heart of the matter—for each well-formed metaphoric expression $\binom{\mathsf{M}}{\phi}$ with halo $\nabla(\mathsf{M}, \phi)$ there are the following two postulates, one an axiom schema, the other a rule:

(ME) $\binom{\mathsf{M}}{\phi} \to \theta$, for all $\nabla(\mathsf{M}, \phi)$. (M-elimination)

(MI) If $\theta \to \varpi$ is a thesis for all $\varpi \in \nabla(\mathsf{M}, \phi)$,
then so is $\theta \to \binom{\mathsf{M}}{\phi}$. (M-introduction)

One might also consider adding as new axiom schemata (assuming that the expressions involved are well-formed):

(M$\neg$) $\binom{\mathsf{M}}{\neg\phi} \leftrightarrow \neg\binom{\mathsf{M}}{\phi}$

(M$\wedge$) $\binom{\mathsf{M}}{\phi\wedge\psi} \leftrightarrow \binom{\mathsf{M}}{\phi} \wedge \binom{\mathsf{M}}{\psi}$

In any case, the theory presented here is an idealization that in some ways fails to represent the intuitions that we have. Let us say (assuming that $\binom{\mathsf{M}}{\phi}$ and $\binom{\mathsf{M}}{\psi}$ are well-formed) that $\binom{\mathsf{M}}{\phi}$ and $\binom{\mathsf{M}}{\psi}$ are *metaphorically equivalent* if and only if $\binom{\mathsf{M}}{\phi} \leftrightarrow \binom{\mathsf{M}}{\psi}$ is a thesis. This means that the sentence

Juliet is the sun

is metaphorically equivalent to each of the sentences

Juliet is the sun, and $2 + 2 = 4$,

Juliet is the sun, and 7 times 142857 equals 999999,

or, more generally, to

Juliet is the sun, and ϕ,

where ϕ is any true arithmetical statement. This is probably counterintuitive, even if it is granted that the only aspect of metaphor that is considered in this paper is the informative one. However, it should be remembered that a similar idealization of the concepts of belief and doxastic commitment has been built into our formal semantics: they are the concepts of belief and doxastic commitment of a "completely rational" agent. For such an agent our concept of metaphor might be appropriate.

2.3. Formal semantics

An aptly chosen metaphor generates understanding thanks to its "halo effect": the fact that (if the metaphor "works") a certain set—above called "the halo"—of ordinary propositions, expressed by literal formulæ, is suggested. Suppose that $\mathfrak{M}$ is a model. In the preceding section it seemed unnecessary to introduce names for models as we were dealing with only one model at a time. But now it becomes important to keep track of models as the very point of metaphor is model expansion: A metaphor is an implicit request for a new, expanded model; a successful metaphor induces one.

So suppose that $\mathfrak{M}$ is a model and that $\binom{\mathsf{M}}{\phi}$ is a metaphoric expression. We would like to define the meaning of $\binom{\mathsf{M}}{\phi}$ in $\mathfrak{M}$, denoting it by $[\![\binom{\mathsf{M}}{\phi}]\!]^{\mathfrak{M}}$. The way to do this seems obvious:

$$[\![\tbinom{\mathsf{M}}{\phi}]\!]^{\mathfrak{M}} =_{\mathrm{df}} \bigcap\{[\![\varpi]\!]^{\mathfrak{M}} : \varpi \in \nabla(\mathsf{M}, \phi)\}.$$

When we are going to carry out this intuitive idea formally, it is should be noted that, just as we have extended the formal language by allowing metaphoric expressions, and just as we have extended the formal syntax, so we must now in a similar way extend the formal semantics. But this is not difficult: essentially it is achieved by adopting the definition just displayed.

Let us say that a valuation *respects* a metaphor $\binom{\mathsf{M}}{\phi}$ if the condition displayed above holds. The problem is that the expression $\binom{\mathsf{M}}{\phi}$, although by definition a formula, is not a propositional formula, and so is not an element of the domain where ordinary truth-value assignment functions are defined. This, one may say, is the very point of metaphor. What is true—and what makes an aptly chosen metaphor intelligible—is that once the value $[\![\varpi]\!]$ is defined for each $\varpi \in \nabla(\mathsf{M}, \phi)$, then it becomes possible to extend a given ordinary truth-value assignment as described above.

The lives and deaths of metaphors

Metaphor is a powerful tool for extending natural language. Someone, trying to convey an insight that seems to be beyond words, may come up with something that, strictly speaking, is nonsense but nevertheless seems somehow pointful: a metaphoric expression which engenders understanding. If the metaphor is successful, it may be used again. It is a reasonable hypothesis that the more often a new metaphor is used, the greater the likelihood that it will be used again—success breeds success. If it is so successful that it gets to be used routinely, then over time language users will gradually cease to think of the once-upon-a-time-metaphor as a metaphor—it has become part of the language. But the language of which it has become part is now, strictly speaking, slightly larger than it was before. One may say that the theme of this paper is the formal representation of this observation.

Some metaphors are still-born in the sense that they are never used again. Others survive, many of them losing their flavour of metaphoricity over time. It is this transition from linguistic misfit to household phrase that we have tried to model in this paper: how an expression with metaphoric meaning in one model, $\mathfrak{M}$, is given a literal meaning in an expanded model $\mathfrak{M}'$. The distinction between live metaphors and dead metaphors can thus be given a precise expression in the semantics presented here: in $\mathfrak{M}$ the metaphor is alive, in $\mathfrak{M}'$ it has died in the sense of having become just one expression among other expressions.

This may sound like a heartless way of describing the life and death of a metaphor. A more merciful formulation would be to say that it has gone on to a new life at a higher level. For really successful metaphors there is life after death.

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References

[1] Alchourrón, C., P. Gärdenfors, and D. Makinson, 'On the logic of theory change: partial meet contraction and revision functions', *The Journal of Symbolic Logic* 50:510–530, 1985.

[2] Gärdenfors, P., *Conceptual spaces: the geometry of thought*, Cambridge University Press, 2000.

[3] Lakoff, G., and M. Johnson, *Metaphors we live by*, Chicago University Press, 1980.

[4] Lindström, S., and K. Segerberg, 'Modal logic and philosophy', in P. Blackburn, J. van Benthem, and F. Wolter (eds.), *Handbook of modal logic*, Elsevier, 2007, pp. 1149–1214.

[5] Quinn, A., *Figures of speech: 60 ways to turn a phrase*, Layton, Utah: Gibbs Smith Publisher, 1982. Reprinted 2010 by Routledge.

[6] Segerberg, K., 'The basic dynamic doxastic logic of AGM', in M.-A. Williams and H. Rott (eds.), *Frontiers in belief revision*, Applied Logic Series, vol. 22. Dordrecht, The Netherlands: Kluwer, 2001, pp. 57–84.

[7] Segerberg, K., 'Iterated belief revision in dynamic doxastic logic', in A. Gupta, R. Parikh, and J. van Benthem (eds.), *Logic at the crossroads: an interdisciplinary view*, New Delhi: Allied Publishers Pvt. Ltd., 2007, pp. 331–343.

[8] Segerberg, K., 'Strategies for belief revision', in J. van Eijck, and R. Verbrugge (eds.), *Games, actions and social software*, Berlin: Springer-Verlag, to appear.

[9] Segerberg, K., 'Some completeness theorems in the dynamic doxastic logic of iterated belief revision', *The Review of Symbolic Logic* 3:228–246, 2010.

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